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A NEW CLASS OF RECTANGULAR DESIGNS

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SUMMARY

A new class of partially balanced incomplete block (PBIB) design based on rectangular association scheme has been obtained through the development of Q_2 Arrays.

Keywords : Partially Balanced Incomplete Block Design, Rectangular Design, and Q_2 Arrays.

Introduction

The rectangular association scheme is a three class association scheme introduced by Vartak [8]. A partially balanced incomplete block (PBIB) design with $v\,(=mn)$ symbols arranged in a rectangle of m rows and n columns follows a rectangular design (RD), if with respect to each symbol, the first associates are the other $n-1$ symbols of the row, the second associates are the other $m-1$ symbols of the same column, and the remaining $(m-1)(n-1)$ symbols are the third associates. For this association scheme, $n_1 = n-1$, $n_2 = n-1$, $n_3 = (m-1)(n-1)$. In fact, a RD is a PBIB design $(mn, b, r, k, \lambda_1, \lambda_2, \lambda_3)$, the symbols having usual significance. The following relations among the parameters hold :

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(i) $mn. r = b. k.$ and (ii) $(n-1) \cdot \lambda_1 + (m-1) \cdot \lambda_2$

$+ (m-1)(n-1) \cdot \lambda_3 = r.(k-1).$

The existing literature reveals that different classes of rectangular designs have been obtained in the following communications, viz. Agarwal [1], [2], Agarwal and Singh [3], Bhagwandas *et al.* [4], Gill [5] and Puri, *et al.* [7].

In this communication a new method of construction of rectangular designs has been developed from Q_2 arrays (definition 1.2).

The following matrix operations will be used in the latter section.

Let $A^{r_1 \times n_1} = (a_{ij})$ and $B^{r_2 \times n_2} = (b_{ij})$ be two matrices whose elements belong to Σ, Σ being a finite module containing s elements.

(1) Let, $r_1 = r_2 = r$, $A \oplus B$ is an $r \times \overline{n_1 n_2}$ matrix where for any column of A, say α_i, and any column of B, say β_j, we define a column $\alpha_i + \beta_j$ of $A \oplus B$, where the location of this column is situated at the $[(i-1) n_2 + j]$ the column $i = 1, 2, \ldots, n_1$; $j = 1. 2, \ldots, n_2$, the symbol $\oplus$ representing the usual vector addition. It is to be noted that the sum of $\alpha_i + \beta_j$ is to be reduced under mod's system.

Definition 1.1 Let, $X = (x_1, x_2, \ldots, x_m)'$ to be a column vector and $Y_1, Y_2, \ldots, Y_m)'$ be another column vector. The symbolic inner product of X and Y is defined by a column vector $X \odot Y = (x_1 y_1, x_2 y_2, \ldots, x_m y_m)'$.

Let $\alpha_i = (\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{it})$ be s^t, t-tuples where $\alpha_{ij} \in \Sigma$, $i = 1, 2, \ldots, s^t$ and $j = 1, 2, \ldots, t$.

Definition 1.2. Two tuples α_i and α_{i1} are said to be independent If $\alpha_{i'} \neq \alpha_s J^{t \times 1} + \alpha_i$ for $(i \neq i')$ and $\alpha_s \in \Sigma$, $J^{t \times 1}$ being a $t \times 1$ matrix with elements unit only.

In case of equality, the two tuples α_i and $\alpha_{i'}$ become mutually dependent. There exists s^{t-1} independent t-tuples in the set of s^t t-tuples.

It is always possible to obtain s^{t-1} different sets of t-tuples, each set containing s-number of mutually dependent t-tuples. The congregation of such s^{t-1} different sets constitute all possible s^t different t-tuples.

Definition 1.3 An array Q_t $(\mu s^{t-1}, r, s)$ containing r rows and s^{t-1} column with elements belonging to Σ, is such that in every t-rowed submatrix of Q_t, μ-number of (distinct of indistinct) t-tuples appear from each set of s^{t-1} different sets.

The Q_t arrays are similar to S_t arrays defined in Mukhopadhyay [6].

2. Method of Construction

The multiplication table constructed with elements from Σ provides a Q_2 (s, s, s), where s is a prime or prime power.

THEOREM. *Let $md = s\text{-}1$ where m and d are both positive integers greater than one (> 1) the existence of Q_2 $(s. s. s.)$ implies the existence of RD $(ms, s(s - 1), d(s - 1), (s - 1), d(d - 1), O, d^2)$,*

Proof: (by construction). Let A be a Q_2 (s, s, s) where s is a prime or prime power and obtained from the multiplication table of s elements, where s $GF(S)$. Since A is obtained from the multiplication table there exist one row and one column of A whose all elements are null. Delete the null row and null column from A, and denote the resulting array by $A'_{[(s-1) \times (s-1)]}$

From $A'^{[(s-1) \times (s-1)]}$ construct the matrix $B_i^{(s-1) \times (s-1)} = \alpha_i J^{(s-1) \times 1} \oplus A'^{(s-1) \times (s-1)}$, $i = 0, 1, 2, \ldots, (s\text{-}1)$, where $\alpha_i \in GF(s)$ and $J^{(s-1) \times 1}$ is an $(s - 1) \times 1$ matrix whose all elements are unit only, in particular $\alpha_0 = 0$ and $B_0 = A'$.

Now obtain,

$$B^{(s-1) \times \overline{s(s-1)}} = [B_0 \mid B_1 \mid B_2 \ldots, \mid B_{s-1}]$$

Let $X = (x_{11}, x_{12}, \ldots, x_{1m}, x_{21}, \ldots, x_{2m}, \ldots, x_{d1}, \ldots, x_{dm})'$ be a column vector, where $x_{ij} = x_{i'j}$ for $i' \neq i'$; $i = 1, 2, \ldots, d$, $j = 1,2, \ldots, m$ and x_{ij}'s are m distinct integers belonging to the set $0, 1, 2, \ldots, m - 1$ for each j, $m < s$.

The symbolic inner product of column vector x with each column of the array B provides $s(s - 1)$ blocks of the RD $(ms, s(s - 1), d(s - 1), (s\text{-}1); d(d - 1)\ 0, d^2)$, where $dm = s - 1$.

Illustration. Take B_0 as :

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 1 & 3 & 5 \\ 3 & 6 & 2 & 5 & 1 & 4 \\ 4 & 1 & 5 & 2 & 6 & 3 \\ 5 & 3 & 1 & 6 & 4 & 2 \\ 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}$$

The following two designs D_1 and D_2) can be obtained by taking the symbolic inner products of $B_0, B_1, \ldots, B_6$ with x_1 and x_2 respectively, where $x_1 = (0, 1, 2, 0, 1, 2)$ and $x_2 = [(0, 1, 0, 1, 0, 1,)$.

The parameters of R D, (D_1 for x_1 are $D_1 : V = 21,\ b = 42,\ r = 12,\ k = 6,\ \lambda_1 = 2,\ \lambda_2 = 0,\ \lambda_3 = 4$ and the parameters of RD, D_2 for x_2 are $D_2 : V = 14,\ b = 42,\ r = 18,\ k = 6,\ \lambda_1 = 6,\ \lambda_2 = 0,\ \lambda_3 = 9$.

$D_1^{6\times 42}$

01	02	03	04	05	06
12	14	16	11	13	15
23	26	22	25	21	24
04	01	05	0[illegible]	06	03
15	13	11	16	14	12
26	25	24	23	22	21

02	03	04	05	06	00
13	15	10	12	14	16
24	20	23	26	22	25
05	02	06	03	00	04
16	14	12	10	15	13
20	26	25	24	23	22

03	04	05	06	00	01
14	16	11	13	15	10
25	21	24	20	23	26
06	03	00	04	01	05
10	15	13	11	16	14
21	20	26	25	24	23

04	05	06	00	01	02
15	10	12	14	16	11
26	22	25	21	24	20
00	04	01	05	02	06
15	16	14	12	10	15
22	21	20	26	25	24

05	06	00	01	02	03
16	11	13	15	10	12
20	23	26	22	25	21
01	05	02	06	03	00
12	10	15	13	11	16
23	22	21	20	26	25

06	00	01	02	03	04
10	12	14	16	11	13
21	24	20	23	26	22
02	06	03	00	04	01
13	11	16	14	12	10
24	23	22	21	20	26

00	01	02	03	04	05
11	13	15	10	12	14
22	25	21	24	20	23
03	00	04	01	05	02
14	12	10	15	13	11
25	24	23	22	21	20

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REFERENCES

[1] Agarwal, K. R. (1975) : Analysis of EGD and hypercubic designs as factorial experiments. *Austral. J. Statist.* **17** 36-42.

[2] Agarwal, K. R. (1977) : Block structure of certain series of EGD and hypercubic designs. *J. Indian Soc. Agril. Statist.* **29**: 19-23.

[3] Agarwal, K. R. and Singh, T. P. (1981) : Methods of construction of balanced arrays with application to factorial designs. *Calcutta Statist. Assoc. Bull.* **30** : 89-93.

[4] Bhagwandas, Banerjee, Kageyama, S. (1985) : Patterned construction of partially balanced incomplete block designs. *Commun. Statist. Theor. Method,* **14** (6), 1259-67.

[5] Gill, P. S. (1986) : Balanced incomplete arrays. *J. Statist. Planning and Inf.,* **14** : 179-185

[6] Mukhopadhyay, A. C. (1981) : Construction of certain series of Orthogonal arrays. *Sankhya (B)* **43** : 81-92.

[7] Puri P. D., Mehta, B. D. Kageyama, S. (1987) : Patterned constructions of partially efficiency-balanced designs. *J. Statist. Plann. and Inf.,* **15** : 65-378..

[8] Vartak, M. N. (1955) : On an application of Kronecker product of matrices to statistical designs. *Ann. Math. Statist.,* **26** : 420-408.